Research project RL

Explicit reciprocity laws for higher dimensional schemes

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Research project short description: Let $V$ be a $p$-adic representation of the absolute Galois group $G_K$ of a $p$-adic field $K$. If $V$ is a de Rham representation in the sense of the $p$-adic Hodge theory Bloch and Kato [2] defined the so-called dual exponential map

$$\exp^*_V : H^1(G_K, V) \to \text{Fil}^0 D_{\text{dR}}(V)$$

with values in the 0th term of the canonical filtration of the de Rham module $D_{\text{dR}}(V)$. Roughly speaking, an explicit reciprocity law for $V$ is an explicit description of $\exp^*_V$ in terms of differential forms and residues. The importance of this question is related to the hope that the images of the so-called special elements of $H^1(G_K, V)$ under $\exp^*_V$ coincide with special values of associated $L$-functions. This was proved by Kato in the following cases:

1) $V = \mathbb{Q}_p(m)$ ($m \geq 1$). Here the special elements are cyclotomic units and the associated $L$-function is the Riemann zeta-function. The main local ingredient is the explicit reciprocity law for $\mathbb{Q}_p(m)$ proved in [3].

2) $V$ is a $p$-adic representation associated to a modular form $f$ on $\Gamma_0(N)$. Here the $L$-function is $L(f, s)$ and the special elements are Kato’s elements constructed using Siegel’s units. The main local ingredient is the explicit reciprocity law constructed in [K2].

The proofs of Kato’s explicit reciprocity laws are based on methods of the $p$-adic Hodge theory. On the other hand, in [1] it is shown that a reciprocity law can be seen as an explicit construction of the $p$-adic periods pairing.

The goal of this project is to generalise the explicit reciprocity law from [4,5] to higher dimensional schemes.
REFERENCES