Research project **ALF**

Algorithms for local fields

Supervisor 1 Hendrik Lenstra
E-mail hwl@math.leidenuniv.nl
Institution Universiteit Leiden

Supervisor 2 Karim Belabas
E-mail Karim.Belabas@math.u-bordeaux1.fr
Institution Université Bordeaux 1

Research project short description:

The researcher will investigate fundamental algorithms related to local fields, and the complexity of those algorithms. Here a local field is a finite extension of a field of the form $\mathbb{Q}_p$ or $\mathbb{F}_p((t))$, where p is a prime number.

Here is a number of algorithmic questions that are worth studying.

(a) Given a local field E and a finite commutative E-algebra A, determine all prime ideals P of A, and for each of them determine the residue class field degree, the ramification index, and the ring of integers of the field extension A/P of E. As a special case, study the complexity of the problem of factoring a separable polynomial in one variable over a local field. Finite non-commutative algebras may also be considered.

(b) Given a finite separable extension $E \subset F$ of local fields, how close can one get to computing the Galois group of its Galois closure in polynomial time? Can one efficiently find the largest tamely ramified subextension of the Galois closure?

(c) Study the complexity of evaluating the reciprocity map from local class field theory as well as its inverse.

There are several ways in which algorithms for local fields differ from the more familiar ones over global fields. For one thing, in the local case one needs to keep track of the precision required in the algorithms. For another, one will, just as with finite fields, often need to be satisfied with probabilistic rather than deterministic algorithms.