Research project **ITSM**

Iwasawa theory for supersingular motives

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Research project short description:

Let E be an elliptic curve over $\mathbb{Q}$. To E on may attach two objects of a rather different nature which contain arithmetic information about E. One is complex analytic the L-function of E and the other is algebraic, the set $E(\mathbb{Q})$ of $\mathbb{Q}$-points of E which has the structure of an abelian group. The conjecture of Birch and Swinnerton-Dyer predicts an important numerical relationship between these two objects: the order of vanishing of L at 1 is the rank of $E(\mathbb{Q})$.

Let us now choose a prime integer p which is a prime of a good ordinary reduction for E (ordinary means that p does not divide the number $a_p = p + 1 - card(E(\mathbb{F}_p)))$. By interpolating the special values at 1 of the L-function of E twisted by characters of p-power order one obtains a power series in one variable with $\mathbb{Z}_p$-coefficients which we call the analytic p-adic L-function of E. On the other hand, one defines an algebraic p-adic object called (dual) p-primary Selmer group of E which is a torison module over the power series ring in one variable with $\mathbb{Z}_p$-coefficients which we usually denote by Λ. A characteristic power series of the Selmer group will then be called an algebraic p-adic L-function of E. Iwasawa's Main conjecture for E states that the algebraic and analytic p-adic L-functions divide one another. This conjecture was recently proved (under some mild hypothesis) by work of Kato [Ka] and Skinner-Urban [S].

If now p is a prime of good supersingular reduction for E (i.e. p divides the number a_p defined above) then the situation is completely different. There are two analytic p-adic L-functions and each one of them is a power series with coefficients in a finite extension of $\mathbb{Q}_p$, in other words the coefficients have (unbounded) denominators. Also the (dual) p-primary Selmer group of E is not a torsion Λ-module so it is not obvious in this situation how to relate the analytic and the algebraic objects and formulate a Main conjecture. If however $a_p = 0$ (this happens whenever $p \geq 5$) R. Pollack [P] (on the analytic side) and S.I. Kobayashi [Ko] (on the algebraic side) showed how one can obtain from these objects analytic and algebraic p-adic L-functions (a couple *of each) which are elements of Λ and which can therefore be compared as in the ordinary case. The work of Kobayashi was recently extended to modular forms by Lei [L]. It would be an interesting project to study algebraic and analytic p-adic L-functions and the respective Main conjectures attached to other "supersingular" motives, for example "supersingular" higher weight modular forms, Hilbert modular forms, Siegel modular forms etc.*

References:

[Ka] Kato, K.: *p-adic Hodge theory and zeta functions of modular forms*, Astérisque.

[Ko] Kobayashi, S.: *Iwasawa theory of elliptic curves at supersingular primes*, Invent. Math. 152 (2003), 1-36.

[L] Lei, A.: *Iwasawa theory of modular forms at supersingular primes*, (Preprint) (2009), 35 pages, available on arXiv:0904.3938v2.

[P] Pollack R.: *On the p-adic L-function of a modular form at a supersingular prime*, Duke Math. J. 118 (2003), 523-558.

[S] Skinner, C.: *Main conjectures and modular forms*, preprint.