Research project **ARFGCCTV**

Amœbas, Ronkin functions, Green currents for cycles on toric varieties

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Research project short description:

The concept of amœba has been introduced by I.N. Gelfand. The amœba of an ideal in $\mathbb{C}[X_1^{\pm 1}, ..., X_n^{\pm 1}]$ *is defined as the image of the zero set of this ideal (in the complex torus* $(\mathbb{C}^*)^n$*) through the map*

$$Log : (z_1, ..., z_n) \mapsto (\log |z_1|, ..., \log |z_n|).$$

An interesting geometric description has been recently proposed by K. Purbhoo [Purb]. The joint work of M. Passare and H. Rullgård enlightens the crucial role played in codimension one by the so-called Ronkin function [Ronk] ; when the hypersurface is defined by a Laurent polynomial with integer coefficients, such function appears as a substitute for the Mahler measure in the expression of the arithmetic height of the associated divisor. Unfortunately, there is at the moment no analog for the Ronkin function in higher codimension. The goal of this thesis is precisely to explore such a direction and try to make explicit, thanks to an appropriate Ronkin function, the arithmetic height (as defined in [BGS]) of arithmetic cycles in the toric setting instead of the projective one ; underlying algebraic complex varieties will now be defined by a collection of Laurent polynomials $P_1, ..., P_m$*, the fan on which the toric variety is built being compatible with all their supports ; one will profit from homogeneous coordinates as well as from harmonic analysis in the toric setting as developped for example in [BTZ]. One will make explicit a Green current, then a closed formula for the height of an arithmetic cycle, in the spirit of [BY]. The Ronkin function will be interpreted as a tool to measure the contribution at archimedian places in the expression of the arithmetic height for an hypersurface in a simplicial toric complete variety, following the work initiated by P. Philippon and M. Sombra in [PS]. The subject needs some familiarity with pluricomplex analytic geometry (currents, residue calculus) as well as with convex analysis (polytopes, toric geometry, real and complex Monge-Ampere equations).*

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