

*Research project HBCAV***Heights of branched coverings of arithmetic varieties**

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Research project short description: *The aim of this research project is to develop and apply results bounding the height in finite coverings of a fixed arithmetic variety with a fixed branch locus. It is to be foreseen that an important tool in the study of such coverings will be (a suitable version of) the arithmetic Riemann-Roch theorem. For such a theorem to apply, however, several technical complications need to be taken care of, one of them being the fact that generic ramification causes the naturally occurring push-forward vector bundles to have only singular hermitian metrics.*

A concrete case of interest is that of coverings of arithmetic surfaces. A good upper bound for the height as mentioned is one of the possible steps in a proof of existence of a polynomial time algorithm for counting points on the reductions of a given surface modulo primes, using étale cohomology. One subproject will be to work on the further development of such an algorithm.

A function field analogue of the problem has been worked out in [2]. From [3] follows an exponential bound for the height of branched covers of the projective line; for the present purposes a polynomial bound is required. A polynomial time algorithm for "counting points", using étale cohomology, on the "motives" associated to level-one normalised Hecke eigenforms is described in the forthcoming book [1]. The present state-of-the-art on the arithmetic Riemann-Roch theorem can be found in [2].

References:

- [1] J.-M. Couveignes, S.J. Edixhoven (eds.): *On the computation of coefficients of a modular form*. To appear in Annals of Mathematics Studies. Preprint arXiv:0605244.
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- [4] H. Gillet, D. Rössler and C. Soulé: *An arithmetic Riemann-Roch theorem in higher degrees*, Ann. Inst. Fourier 58 (2008), 2169–2189.